



Common Core State Standards for Mathematics: GRADE 3
THE PRIVATE EYE® — (5X) LOOKING / THINKING BY ANALOGY® Correlation



Grade 3

The Private Eye® aligned with Common Core State Standards
for Mathematical Practice and Content



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Welcome!

The Private Eye makes math a language to love, even as it helps build a mathematical vocabulary. It turns math into something personal, intriguing, friendly, alive. The Private Eye's hands-on, interdisciplinary process and instructional strategy braids together three languages: verbal, visual, and mathematical.

The Private Eye begins with four simple questions, everyday objects, and a jeweler's loupe (an almost magical magnification tool). Using The Private Eye process students enhance concentration, heighten their awareness of pattern and detail, and learn to evoke analogic thinking for problem-solving. TPE delivers students directly to the "land of Math" — the science of patterns and relationships. Whenever you use The Private Eye, pattern is "in your face"—you're massaging the math brain, even as you massage the scientist's, writer's, artist's brain.

The Private Eye blends with your existing math course-of-study to develop habits of mind essential to mathematical practice. As you consider your math year, you'll find you can use TPE in your classroom to: introduce, enhance, cement and assess mathematical concepts and content. It helps students settle down and focus as preparation for a new or existing mathematical concept. It calms their fears that a math topic will be too difficult, too foreign. It grounds concepts in a student's personal knowledge and associations and in the five senses. It generates mathematical inquiries that live and breathe.

With its simple tools, rich questioning strategy, and everyday objects, students can write, draw, theorize, count, measure, compute, calculate, estimate, predict and perform mathematical operations. In the process they build four underlying *and interwoven habits of mind* critical to academic success: looking closely, thinking by analogy, changing scale, and theorizing. These are the intellectual "tools" not only for mathematical literacy, but for creativity, literacy, and scientific literacy as well. The book, *The Private Eye —(5X) Looking/Thinking by Analogy: A Guide to Developing the Interdisciplinary Mind*, shows how to fluently develop these essential habits. A special Math Tour of lesson connections begins on page 173.

* * *

This document correlates *The Private Eye—(5X) Looking/Thinking by Analogy* to the mathematical practices and content outlined in the Common Core State Standards for Grade 3. Along the way, the document provides many "how to" examples for meeting and practicing the content of each standard in the context of Private Eye use. (For Private Eye CCSS Literacy correlations, please see our separate publication.)



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Standards for Mathematical Practice		<i>THE PRIVATE EYE® — (5X) LOOKING / THINKING BY ANALOGY®</i> <i>A Guide to Developing the Interdisciplinary Mind</i>	
Meet all eight CCSS Standards for Mathematical Practice using The Private Eye (TPE) in Math <i>and</i> across your curriculum day:			
<p>1. Make sense of problems and persevere in solving them. Thinking by Analogy (making associations and using them for theorizing, inferring, modeling) is how we make sense of the world. The Private Eye boosts analogic reasoning as it also builds concentration, ever-increasing attention to detail, and wonder at the world's patterns and relationships. Using TPE builds perseverance incrementally and naturally. The Private Eye's exploratory inquiry generates options to approach problem solving. It builds <i>problem-solving by analogy</i> using verbal, visual, and mathematical languages.</p>	<p>3. Construct viable arguments and critique the reasoning of others. TPE's inquiry approach includes: "Why is it like that?" "What's going on here?" TPE gives students a hypothesizing and theorizing strategy to answer these questions using words, numbers, images to generate models. Students work individually and collaboratively, examining and critiquing each other's methods and conclusions.</p>	<p>6. Attend to precision. The Private Eye's loupe and questioning strategy gives students a jolt of attention to detail: it literally "teaches" what attention to detail and precision <i>means</i>. Using TPE students explore real world shapes, structures and relationships in conjunction with analogic observations — verbal, visual, mathematical — to express ever more precise communication / thinking. TPE hones ability to discern and distinguish less obvious similarities and differences.</p>	
<p>2. Reason abstractly and quantitatively. Analogic reasoning is the abstract reasoning at the heart of mathematical reasoning. TPE process is rooted in analogic reasoning: proportional reasoning, making inferences, theorizing — based on strategic use of associations. (BTW, the term "analogy" was originally a math term!)</p> <p>"Analogy is the Interstate Freeway of Cognition", notes Douglas Hofstadter. We use analogy in forms verbal, visual, and numerical — creating and using analogs, comparisons, and models — to understand and solve problems. TPE tools and strategy evoke and constantly build analogic / comparative thinking for students and adults. Hands-on explorations quicken abstract reasoning while keeping students grounded in real world applications. Repetition with TPE process makes analogic reasoning in verbal, visual and mathematical languages into a habit for students, an instinctive practice.</p>	<p>4. Model with mathematics. Models are essentially analogies: an exploration and a representation of patterns, structures, behaviors, and relationships we discover in the world <i>around us</i>. Numbers are analogs that explore and represent specific quantities, interactions, operations, measurements, behaviors, and relationships. TPE helps students practice moving between modeling with mathematical analogs and modeling with verbal, visual, and structural analogs.</p>	<p>7. Look for and make use of structure. TPE loupes and Questions help students habitually look closely for structures, patterns and relationships at changes of scale, small and large. In a math context, this habit of mind translates into a heightened sensitivity to numerical structures and sequences.</p>	
<p>5. Use appropriate tools strategically. The Private Eye Tools: a 5X Loupe (a marvelous magnification tool), everyday objects (manipulatives), loupe-drawing, and loupe-analogy observations are all mathematical tools in the context of math explorations. (TPE Questions are tools, as well!) The loupe allows students to change scale — to find mathematical numbers, shapes, concepts and relationships in small places in comparison with large scale situations. The loupe enlarges objects or parts of objects by 5X (10X if two loupes nested) creating a heightened interest in structures, patterns and measurements. It boosts Mathematical Practices #6, 7, and 8.</p>		<p>8. Look for and express regularity in repeated reasoning. Thinking by Analogy fueled with Looking Closely is fundamental to pattern recognition. Using TPE in math — a repeating loop of questions for investigating and reasoning — sensitizes students to looking for regularity in mathematical structures.</p>	
		<p>☒ When you use The Private Eye's interdisciplinary process in math, you not only meet math standards correlated to the lesson, but specific Science and Literacy Standards as well. See CCSS Literacy / TPE correlations.</p>	



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Standards for Mathematical Content	<p align="center">THE PRIVATE EYE® — (5X) LOOKING / THINKING BY ANALOGY®</p> <p align="center"><i>A Guide to Developing the Interdisciplinary Mind</i></p>
<p align="center">GRADE 3</p>	<p>Prep: Read “Process and Tools”, pp. 11-31; and “Math Tour”, pp.175-194. Introduce students to <i>The Private Eye (TPE)</i> loupe & process: looking closely, thinking by analogy, changing scale, and theorizing. TPE’s holistic math activities connect to multiple standards and can be adapted for any grade. (Along the way, it’s easy and fun to create your own lessons.)</p> <p>Practice: In the correlations we provide some “unpacking” of standards, grade-level examples, and a starter list of TPE book connections. On some days, you may choose merely to use the loupe with everyday objects for knock-your-socks-off, content-rich manipulatives to meet detailed elements of a standard. But connect these experiences to a Private Eye-listed activity — even better, to the full, interdisciplinary Private Eye process (TPE Questions / loupe-drawing / writing / science content / theorizing) — and you’ll see a real difference in how students think and communicate mathematically. Students will rev up creative and critical thinking in tandem with math skills. They’ll fall in love with math.</p>
<p>Operations and Algebraic Thinking 3.OA</p> <p>Represent and solve problems involving multiplication and division.</p> <p>3.OA.1 1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i></p>	<p>Represent and solve problems involving multiplication and division.</p> <p>Overview: As students loupe-study, draw, and manipulate multiples of loupe-study objects into various repeating groups (sets) — beans, seeds, pennies, popcorn kernels, leaves, small twigs, etc.— they investigate first-hand the real meaning of multiplication and division. They quickly come to understand and can verbally interpret the meaning of the numbers in a multiplication problem, and can see how multiplication relates to division. Students will be able to explain contexts: that multiplication is actually repeated addition; that the 1st number in a multiplication problem represents the number of times a group (set) is repeated and the 2nd number indicates the number of objects in each group (set). Students move into solving multiplication and division problems and creating them for their peers to solve. They become fluent in abstract thinking when grounded in concrete realities.</p> <p>1. Students interpret products of whole numbers using loupe-study objects to manipulate as multiplication contexts, e.g., they interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>Students are able to describe a context in which a total number of objects can be expressed as 5×7.</i></p> <p>Example: Popcorn Multiplication. After loupe-observing one piece of popcorn and using TPE first questions for bonding with the subject, students receive 35 pieces of popcorn each. The teacher instructs: How many pieces total do you have? Now, arrange your popcorn pieces into 5 groups of 7 pieces each. After discussing interpretations of what each student has created, students represent it by writing $5 \times 7 = 35$ in their Math Journals, in a section they entitle, “Popcorn Multiplication”. Students also write an explanation of the equation, its <i>meaning</i>. Thus they investigate how a total number — in this case, “35” (pieces of popcorn) — represents the “product” of an interaction between two other whole</p> <p align="right">(continued next page...)</p>



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Represent and solve problems involving multiplication and division (continued)

3.OA.1 ... continued

3.OA.2

2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *E.g., describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*

Represent and solve problems involving multiplication and division. (continued)

numbers and can be expressed as: “5 × 7” with the center ‘x’ meaning “groups of”. Students practice visually (and kinesthetically) representing multiplication problems the teacher supplies by manipulating multiples of various loupe-study objects. Conversely, students can begin with manipulating their loupe-study objects into various equal groupings, representing each set of groups with the appropriate equation. Students write the numerical representations and the product / total. One team can challenge another team to represent its physical model. Again, students describe their thinking and extend it to other objects / contexts. Soon students are ready to move into describing and interpreting multiplication problems unlinked to objects at hand.

- Math Plans & Ticklers: Sequence, p. 175
- Seeds! p.180
- Foxglove Towers, p. 180
- Seed Pods Pop p. 145
- Seed Collections, p. 144

2. Students learn to interpret whole-number quotients of whole numbers, using loupe-study objects, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

Example: Beans, Quotients and Division. Students manipulate multiples of loupe-study objects, e.g., pinto beans, to investigate the meaning of whole number quotients of whole numbers in division. As usual, students loupe-study one pinto bean first (a pinto bean is like a little abstract artwork under a loupe!); students use TPE first questions for more precise observation than is possible with the unaided eye, for evoking analogic thinking, and for bonding with an object of study — all of which makes learning each new conceptual stage in math more comfortable and easier to assimilate.

Materials: Students need a bag or cup of 56 beans and a mat to work on. The teacher writes on the board: $56 \div 8$ and asks: If you partition your beans into 8 equal groups — or shares — how many beans are in each share / group? Let students discover the answer, i.e., “the quotient”. This gives students an initial concrete example of a *context in which a number of shares or a number of groups can be expressed as $56 \div 8$. The number of shares in 8 equal groups is 7.*

Students practice representing other such quotient / division problems with other loupe-study objects in multiples (small and large), recording the equations in a Math Journal, and writing down explanations of what’s going on in division.

- Math Plans & Ticklers: Sequence, p. 175
- Seeds! p.180 (Skip the microscope if not available)
- Foxglove Towers, p. 180
- Seed Pods Pop p. 145
- Seed Collections, p. 144



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Represent and solve problems involving multiplication and division (continued)

3.OA.3

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.4

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$*

Represent and solve problems involving multiplication and division. (continued)

3. Either as stand-alone math inquiries or as a math inquiry linked to a science, literature, art, and/or social studies investigations involving loupe-study objects (e.g., seed pods in a plant reproduction study) and The Private Eye process, **students solve word problems within 100 using multiplication and division** by using drawings and equations with a symbol for the unknown number to represent the problem. The word problems should involve equal groups, arrays, and measurement quantities. Both teacher and students create one-step word problems connected to a loupe-study object to solve. This keeps alive the concept that math is rooted in the real-world which helps students really value math. It also makes mathematical problems more personal and intriguing.

Example of teacher generated problem: We're going to be investigating cork, its structure, what it's made of, where it comes from, what it's used for, and how to recycle it. We'll use TPE process to magnify our investigation. I need to put in a supply order now. If I want to insure that every student in the class (24) has 4 corks to study, how many should I order from the catalog?

$$24 \times 4 = \square \qquad 24 \times 4 = 96 \qquad \square = 96$$

Or, the problem as division: I have 96 corks for the class. We have 24 members of the class. If we make up bags of corks and distribute them equally, how many corks will be in each bag?

$$96 \div 24 = \square \qquad 96 \div 24 = \square \qquad \square = 4$$

Example of a word problem connected to an investigation: How many seeds are in the seed pod? If the seed pod has 8 segments and we find that each segment or chamber typically has 11 seeds, how many seeds are in a typical pod? Students write down the word problem in their math journal and the math symbols representing problem, unknown, solution: $\square = 8 \times 11$ $8 \times 11 = 88$ $\square = 88$

- Math Plans & Ticklers: Sequence, p. 175
- Seeds! p.180
- Foxglove Towers, p. 180
- Seed Pods Pop p. 145
- Seed Collections. p. 144

4. Students use loupe-study objects in multiples as concrete models to help **determine the unknown whole number in a multiplication or division equation relating three whole numbers.** *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$*

Example: Peter has a seed pod with 5 chambers. There are 7 seeds in each chamber. How many seeds does Peter have? $5 \times 7 = ?$

Tara has 84 crackers in a box. There are 28 members of her class and she wants to give an equal share to each. How many should each classmate receive? $28 = 84 \div ?$

- Math Plans & Ticklers: Sequence, p. 175
- Seeds! p.180
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- Seed Pods Pop p. 145
- Seed Collections. p. 144



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Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5

5. Apply properties of operations as strategies to multiply and divide.*
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

* Students need not use formal terms for these properties.

3.OA.6

6. Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Understand properties of multiplication and the relationship between multiplication and division.

Overview: Again, you can speed student thinking / understanding of the various relationships between multiplication and division (and deepen the mathematical practices at the same time) if you ground students in concrete models. Have students use multiples of loupe-study objects (e.g., beans, seeds, pennies, leaves), including objects that have many parts, to loupe-observe, loupe-draw, manipulate, multiply and divide into groups, including arrays.

5. Teach students the **properties of operations (how numbers work) as strategies to multiply and divide** by initially using loupe-study objects. Such grounding helps students to “see” and “feel” the logic behind properties of operations visually and kinesthetically, which speeds understanding. This “prelude” helps them move more smoothly into working with abstract numbers only.

Example: Commutative property (informally the “order property”). Students have 15 acorns each. To represent 5×3 ... students arrange acorns into groups or rows (arrays): 5 rows of 3. Represent as equation: $5 \times 3 = 15$. Rearrange into 3 rows of 5 acorns and you still have 15 acorns: $3 \times 5 = 15$. Logic: If $5 \times 3 = 15$ is known, then $3 \times 5 = 15$ is also known.

Example: Associative property. Arrange and rearrange pinto beans to represent an equation with 3 or more whole numbers in a multiplication string. Pair pinto beans with the equations and demonstrate that you can solve such a problem using the associative property: $3 \times 5 \times 2$ can be found if you solve part of the problem first by simplifying it into a smaller pair or “association”: $3 \times 5 = 15$; then continue to multiply the first product by the remaining number, which is 2, i.e., $15 \times 2 = 30$. Now demonstrate that you can solve the larger problem by solving a smaller associated pair in any order and you still end up with the same product/total: $5 \times 2 = 10$, then $3 \times 10 = 30$.

Example: Distributive property (Informally: “breaking numbers apart”). Again, use pinto beans (or some other loupe-study object) so each student can witness up close and personal how this property works to make solving problems easier: Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$.

6. **Students learn to understand division as an unknown-factor problem** initially linked to loupe-study objects. Because many students are not yet fluent in mentally multiplying and dividing within 100, they can also create *proofs* for problems they think they’ve solved using their loupe-study manipulatives.

Example: Instruct students that they can solve division problems if they know the answer to a multiplication problem, without mentally or physically dividing. Sample: $32 \div 8 \dots 8 \times ? = 32?$

Students create *proofs* using concrete loupe-study objects. Give students 32 packing peanuts (Yes! interesting under a loupe!). Divide them into 8 equal groups. Result — Yes! 4 in each group. Apply and practice these skills in lessons below.

- Math Plans & Ticklers: Sequence, p. 175
- Seed Pods Pop, p. 145
- Foxglove Towers p. 180
- Seeds! p.180 (skip the microscope if unavailable)



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Multiply and divide within 100.

3.OA.7

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Multiply and divide within 100.

7. Students build and reinforce fluency in multiplying and dividing within 100 — by connecting the abstract side of the math practice to sensory-rich, practical applications: to the investigation of loupe-study objects that bridge to the content ecosystem in your classroom. Investigating loupe-study objects, students will **use strategies such as the relationship between multiplication and division or properties of operations** (knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) **to generate and solve mathematical questions and problems across the four operations**. When students know from memory all products of two one-digit numbers — which many students will well before the end of Grade 3 but all students should have by year's end — it makes mathematical-scientific investigations even more fruitful and fun.

For working on this standard, it's a great time to advance to going up-close and personal with objects that have *multiple parts*, counting the parts, considering the geometry of the parts — blended with on-going attention to the form-function link between parts and the whole. This holistic approach makes math more practical and meaningful — for the short term and the long run. One approach: Each student's job is to figure out how many such parts would be represented in a given larger group — a task that involves fluent multiplication.

Example: Students are loupe-exploring flowers with 5 petals. They are studying “Forget Me Nots” — a small blue flower, about the size of a fingernail (and so lovely under a loupe!). “Forget Me Nots” bloom prolifically in spring, are popular in gardens, and also grow wild. It's time to link math to the investigation (at the 3rd grade level). Students have already measured the bloom, stem, leaves. They planted some in the school garden. Now they will practice fluently multiplying and dividing with 100. The teacher creates a model word problem for students to solve (below).

Teacher Model: In a small vase you've placed 8 flowers. After a week the petals begin to fall off the flower. If each flower has 5 petals, and if all the petals from those forget-me-nots fall, how many petals will you have to clean up? Quick! Who can tell me the answer? ($8 \times 5 = 40$)

Knowing $8 \times 5 = 40$ allows you to solve the inverse of the problem above. If you didn't know how many petals each flower had, and came upon a table covered with 40 petals, and you found 8 empty stems in a vase, if each flower had the same number of petals, can you figure out how many petals each flower had?

Challenge students to generate novel multiplication and division word problems for each other to solve based on the flower (or other loupe-study objects). Ideally students work in teams of two to create an original problem, determine a solution, write it down in their math journals, and swap problems (no answers provided!) with another team to solve. Or exchange a whole batch of problems with another 3rd grade class to solve. Such work gives students a chance to demonstrate fluency or continue to build fluency in multiplication / division connected to real world investigations.

- Math Plans & Ticklers: Sequence, p. 175
- The Ultimate Portrait p. 177 – 178
- 36 Portraits with Statistics, p. 178
- Estimation / Prediction: Dandelion Math, p. 178 – 179



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Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Students **solve and also generate two-step word problems using the four operations** — linked to explorations of loupe-study objects that relate to your content ecosystem in science, literacy and/or art. Students **represent these problems using equations with a letter standing for the unknown quantity**. The teacher creates the problems at first, as a model. Students then generate two-step word problems and matching equations to represent questions they have about a loupe-study object they are investigating with the help of math. Students also create two-step word problem challenges for peers to solve using the four operations, equation-making, and estimation strategies, including rounding. Once students have learned the meaning of *estimation* and how to do it, **they also practice using mental computation and estimation strategies, including rounding, during mathematical investigations of loupe-study objects, and assess the reasonableness of their answers.**

Example: We discovered butterfly eggs on the underside of leaves in our wildlife garden. A count shows that the leaves have 9 eggs (typically) attached to the underside. If we find 8 leaves with eggs on our first outing, and 6 more leaves the second time, how many eggs in all have we found?

$$(8 \times 9 = m) + (6 \times 9 = n) = y \quad \text{thus } m + n = y \quad 72 + 54 = 126 \quad y = 126$$

Example of two-step problem involving estimation: The class has been loupe-examining strawberries, the outside — and are ready to journey inside the fruit. Each student has his own strawberry. Direct students to cut their strawberry in half (using a plastic knife) and loupe-explore the cut section. After the usual loupe-analogy list, have students cut the half in half; thus the strawberry is cut into fourths (approximately).

Pose this question / problem to students: How many seeds on your whole strawberry? How could we find out, not exactly, but approximately? Discuss with students how estimating saves time but can still be close to accurate. Have each student count the seeds on the outside (achenes which hold the seeds) and record the number on in math journal.

Lead students to discover an estimation strategy: that if they know how many seeds are on 1 of 4 roughly equal parts, they can use addition or multiplication that involves rounding to solve the problem. Ask: Once you have a fourth of the seeds, will this help us estimate the total number on the strawberry? How to represent the problem with an equation?

Let x = total number of seeds Let s = number of seeds on a *fourth* of the strawberry

$$x = s + s + s + s \quad \text{or} \quad x = 4 \times s \quad \text{or}$$

[\(continued next page...\)](#)



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Solve problems involving the four operations, and identify and explain patterns in arithmetic. (continued)

3.OA.8 (continued)

3.OA.9

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Solve problems involving the four operations, and identify and explain patterns in arithmetic. (continued)

Example: The class just learned, as well, that strawberries have large amounts of Vitamin C and Vitamin C is good for everyone’s health. To increase her Vitamin C intake, Jane decides to eat 5 strawberries a day. Her goal is to eat 60 strawberries. After 5 days, how many strawberries does Jane still have to eat in order to meet her goal? Write an equation:

$(5 \times 5) + s = 60 \quad \dots \quad 25 + s = 60 \quad \dots \quad 60 - 25 = s \quad \quad (s = 35 \text{ more strawberries})$

Example involving estimation: The class eats 120 strawberries the 1st week of May. 197 strawberries the 2nd week, and 303 strawberries the 3rd week. Estimate how many strawberries the class ate, using rounding.

- Math Plans & Ticklers: Sequence, p. 175
- Estimation / Prediction ... Dandelion Math: How Many Seeds? p. 178 - 179
- The Ultimate Portrait p. 177 – 178
- A Game Like: “How Many Marbles in This Jar?”, p. 180 (Use for any flower seedpod!)
- Seeds! p. 180 (Use for any flower seedpod! Skip the microscope if unavailable.)
- Map a Strawberry and count / estimate sword fern spores (two more great objects of study)



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Numbers and Operations in Base Ten 3.NBT	
<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>3.NBT.1</p> <p>1. Use place value understanding to round whole numbers to the nearest 10 or 100.</p> <p>3.NBT.2</p> <p>2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>3.NBT.3</p> <p>3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>3.1, 3.2, and 3.3: Students complement their Private Eye close-up encounters (e.g., peppercorns, beans, popped popcorn, daisies, etc.) with adventures in place value, combined with operations performing multi-digit arithmetic. They extend their understanding of place value and the properties of operations as they practice fluently adding and subtracting within 1000, and solve multi-digit arithmetic problems to investigate loupe-study objects and the broader topics with which they connect.</p> <p>Note: For students who are on shaky ground in understanding of place value through 1000, see the Place Value Activity in Grade 2 Private Eye correlations: 2.NBT.1 “Pinto Bean Place Value”, p. 3</p> <p>Example: Students have been loupe-observing peppercorns as part of studying the Spice Trade, cooking, and nutrition. Now they extend their peppercorn study to math. The first problems are generated by the teacher (below). Students then create novel word problems based on peppercorns for peers to solve.</p> <p>Robert has three jars of peppercorns. The first jar has 148 peppercorns. The second has 181, the 3rd has 152. Have students use place value reasoning to figure the total.</p> <p>The soup recipe calls for 9 peppercorns per cup of soup. The cook has to make 80 cups of soup for the cafeteria. How many peppercorns does the cook need? (Have students solve the problem using multiplication of a one digit number with multiples of 10. Have students elaborate their reasoning.)</p> <p>Example: Students have been loupe-observing and sprouting lima beans.</p> <p>Ellen has 4 small bags of lima beans. Each bag contains 50 beans. Figure the total number of beans. (Have students solve the problem using multiplication of a one digit number with multiples of 10. Have students elaborate their reasoning.)</p> <ul style="list-style-type: none">• Math Plans & Ticklers: Sequence, p. 175• Estimation / Prediction ... Dandelion Math: How Many Seeds? p. 178 - 179• The Ultimate Portrait p. 177 – 178• Seeds! p. 180 (Use for any flower seedpod! Skip the microscope if unavailable.)• Foxglove Towers, p. 180 (Use for any flower seedpod!)



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Number and Operations—Fractions	3.NF
<p>Develop understanding of fractions as numbers.</p> <p>3.NF.1 1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>3.NF.2 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p>	<p>Develop understanding of fractions as numbers.</p> <p>Overview: Ground the concept of fractions in the real world — at a desktop scale — and develop a better understanding of fractions. Use loupe-study objects for investigating fractions (e.g., beans, seeds, etc.) to visually arrange into fractional groups; or use objects that have obvious parts that can be separated or cut into real-world fractions (flowers with petals that can be removed and manipulated) or use objects that can be cut in halves, quarters, etc., (apples, oranges, a piece of bread; a swatch of mesh fabric with an interesting weave; even paper, since paper comes from wood/trees and torn paper is interesting to loupe-study!). Use these and many more objects (see pp. 70-71 “What to Collect”) to teach fractions and their numerical representations, including representation on a number line.</p> <p>3.1. and 3.2. Students use loupe-study objects (as noted above) to help understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. After they have practiced creating physical fractions, show them how to represent these physical realities on a number line.</p> <p>Example: “Eat your Fractions!” Demonstration / Cut one apple into 8 parts. The teacher holds up one slice: “This is one part of the whole apple, or a <i>fraction</i> of the apple. What part or <i>fraction</i> of the apple is it? Well, expressed as a fraction, it’s: $1/b$ and since ‘b’ = 8... in this case one slice is $1/8$ th of the apple.”</p> <p>The “a” or top number of a fraction (the numerator) identifies a particular quantity (number) of parts of the whole.</p> <p>a/b = the quantity formed by however many parts of something — in this case an apple — you want to identify or do something with. If you want to eat or give away or mash 5 parts, that’s _____ how many parts of the apple? What fraction of the whole apple? = $5/8$ths</p> <p>Do this with any number of objects students are loupe-studying and, where possible, link to larger studies. E.g., Perhaps students are studying fruits, apples in particular. Combine a study of fractions to cutting an apple into fractions and loupe-exploring the flesh, seeds (also an opportunity for working with fractions!), apple flowers, economics of apple industry, etc. There are lots of worthy fruits, vegetables and nuts to use for such fractional studies. Students can eat the fractions at the end!</p> <p>Now represent your fractional studies (and eatings!) on a number line.</p> <ul style="list-style-type: none"> • Math Plans & Ticklers: Sequence, p. 175 • The Ultimate Portrait, p. 177 – 178 (Use for a daisy, for example, instead of a dragonfly) • Dandelion Math: How Many Seeds, p. 178 • The Incredible Shrinking You, p. 182 (Instead of starting with 1 inch, start with 48” and begin shrinking by incremental fractions)



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Develop understanding of fractions as numbers. (con't)

3.NF.3

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Develop understanding of fractions as numbers. (continued)

3. Use loupe-study objects to help **students understand and explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.**

“Equivalent” means two things are equal, or the same, in a particular respect: size, shape, force, amount, value, meaning, etc. Here, the focus is size. In The Private Eye guide, read: “Analogy Anatomy” p. 42 — for an understanding of equivalences expressed both in words and in numbers and how the two are related.

a. Students **use loupe-study objects as models for fractional equivalencies to understand two fractions as equivalent** (equal) if they are the same size, or the same point on a number line.

Example: Give each student a set of 12 beans in a cup and a white paper mat with an = sign in the middle. Ask: How many parts in the whole amount? Yes, 12 parts in the whole amount. What fraction is 1 part? Yes, one bean is 1 of 12 parts, thus represented as: $1/12$. (Students write their fractions onto an index card or in a Math Journal.) Now, divide whole group of beans *in half*, into 2 equal parts: How many in each part? Yes, 6. Represent as a fraction: $6/12$. Line up half the beans on one side of the = sign and the remaining half on the other side. Represented how? Yes: $6/12 = 6/12$ but also $1/2 = 6/12$. Let it sink in. Go further: Divide each of those 2 parts (of 6 beans each) into 2 equal parts. How would we represent this? What other ways could we represent this? And so on.

Link such physical models to the written representation of the fractions and to number line representations. Progress from simpler objects and parts to more complex to underscore that math is everywhere in the real world.

b. Students use loupe-study objects to **recognize and generate simple equivalent fractions**, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Students **explain why the fractions are equivalent, e.g., by using loupe-study objects as a visual fraction model.**

Example: Students use loupe-study objects for fractions as noted in Overview of this Standard Cluster. Have students create various fractional arrangements..., then link to the written representation of the fractions. Progress from simpler objects and parts to more complex, and to equivalences. You can teach how 1 is related to 2 in the same way that 2 is related to 4 (an analogy in mathematics!) — and the written fraction equations to express this reality.

- “Analogy Anatomy” p. 42 — for an understanding of equivalences expressed both in words and in numbers
- Math Plans & Ticklers: Sequence, p. 175
- The Ultimate Portrait, p. 177 – 178 (Use for a daisy, for example, instead of a dragonfly)
- Dandelion Math: Part I, p. 178
- (Math + Writing) The Incredible Shrinking You, p. 182 (Instead of starting with 1 inch, start with 48” and begin shrinking by fractions)



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Represent and interpret data.

3.MD.3

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Represent and interpret data.

3. Data represented on scaled picture graphs and bar graphs is a kind of collective snapshot, story or profile. It helps us get a bigger picture of events or frequencies or amounts of things — from occurrence of emotional feelings to favorite pets, from variation in a given species to occurrence and variation of types of weeds in neighborhood. Once students are familiar with drawing and using scaled picture and bar graphs, it’s a great time to begin using graphs to create a bigger picture about some loupe-study object that represents a wider or deeper issue. **Students and teacher figure out what they want to represent on a graph (involving their loupe-study object/topic) then draw a scaled picture graph and a scaled bar graph to represent that data set with several categories.**

Students go on to solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

Example: Fabulous Fingerprints! Many teachers have students do the fingerprint loupe-drawing lesson described in “The Simple Touch” on p. 136 -137. Link the artistic interpretation of a fingerprint to a fingerprint classification and identification activity then tally and chart the occurrence of the three main types of fingerprints distributed in one class. There are three main kinds of fingerprints: whorls, loops, waves. For a more advanced version of the activity: have students make a graphite print of each finger on one hand. Some people have up to 3 kinds of prints on one hand! Now the picture graph or bar graph will have even more data to represent.

Example: Bean Soup: The teacher purchases bags of beans — 3 kinds: lima, black, pinto — and mixes them up randomly in a big bowl for a “bean soup mix”. Give each student 1/4 C of beans to sort, count types, and create their own picture graph, then bar graph, to represent how many of each type of bean in their soup mix. Then solve problems by having students compare their results with a partner and express similarities and differences in sentences and with symbols: “how many more” or “how many less” of each bean type one student has compared to another’s mix.

Example: There are so many characteristics of natural objects for students to discover and identify as they loupe-travel! Choose one or more characteristics and graph the frequency, number, variation in color or shape or size (etc.) on simple scaled picture or bar graphs or, in older grades, more complex graphs.

- The Simple Touch, p. 136
- Variation in Nature: A Dandelion’s Fingerprint
- The Ultimate Portrait, p. 177 – 178
- 36 Portraits with Statistics p. 178,
- Foxglove Towers, p. 180.



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<p>Represent and interpret data.</p> <p>3.MD.4</p> <p>4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.</p>	<p>Represent and interpret data.</p> <p>4. Students generate measurement data by measuring lengths of loupe-study objects (beans, twigs, leaves, flowers, petals, stamps, coins, fruits, vegetables, lengths of rope or yarn, etc.) using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units: whole numbers, halves, or quarters.</p> <ul style="list-style-type: none"> • Thoreau’s Backpack and a Tape Measure, p. 177 • Tape Measure / Ruler, p. 177 • The Ultimate Portrait, p. 177 – 178 • 36 Portraits with Statistics p. 178 • Foxglove Towers, p. 180
<p>Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</p> <p>3.MD.5</p> <p>5. Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p> <p>3.MD.6</p> <p>6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p>	<p>Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</p> <p>5. Students recognize area as an attribute of plane figures and understand concepts of area measurement as it applies to loupe-study investigations. Students measure areas by counting the unit squares involved in a loupe-study investigation.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. (Examples included under #6 below.)</p> <p>6. Measure areas by counting unit squares (square cm, square m, square in, square ft, improvised units).</p> <p>Examples: Students identify, mark and loupe-investigate a square inch or square centimeter of a loupe-study object (sponge, slice of bread, newsprint, the back of their own hands, or any object large enough to have “a unit square” within it. Students can move to objects with larger areas that have smaller worlds within worlds. They can mark, map and investigate outdoor habitats — even a section of an brick wall (not only is the material of brick and concrete interesting, but there are often small critters making their homes in such locations, or mosses and lichens!) — or investigate square units of school gardens or trunks of trees, or other habitats, marking one “unit square”: square cm, square m, square in, square ft, and improvised units.</p> <ul style="list-style-type: none"> • A Yard of Yard, p. 148, and Habitats / Systems (observing one square foot or one square meter or of an ecosystem and smaller units within) • A Plot of Grass, p. 148 (observing 1 square foot or 1 square meter and smaller units within) • Traveling a Mathematical Loupe Area and Perimeter p. 176 • Thoreau’s Backpack and a Tape Measure, p. 177 • Your Interdisciplinary Hand, p. 84 • The Simple Touch, p. 136 (For an enlarged fingerprint: use unit squares (square cm, square m, square in, square ft, and improvised units) to map out and measure the area covered by the fingerprint.)



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<p>3.MD.7 7. Relate area to the operations of multiplication and addition.</p>	
<p>Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</p> <p>3.MD.8 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>	<p>Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</p> <p>8. Students solve real world and mathematical problems involving perimeters of polygons connected to loupe-study investigations. Students extend this to irregular polygons and shapes both natural and manmade. Loupe-explorations involving perimeters of polygons can focus on objects, small or large, and can involve objects in locations, indoors or out, such as a section of an ecosystem or habitat.</p> <p>Students identify, mark, measure and infer similarities and differences in the perimeters of such manmade and natural loupe-study objects as: bread slices, kitchen sponges, newsprint sections, paper towels, corrugated cardboard box sections, indoor or outdoor habitat studies, etc. Students can extend this to irregular polygons and shapes both natural and manmade.</p> <p>Begin with problems designed first by the teacher, and then by students for their peers to solve. Students will explore and solve problems that include:</p> <ul style="list-style-type: none"> <input type="checkbox"/> finding the perimeter if given the side lengths <input type="checkbox"/> finding an unknown side length <input type="checkbox"/> rectangles with the same perimeter and different areas <input type="checkbox"/> rectangles with the same area and different perimeters <p>Example: If each side of a slice of bread is 5 inches, what is the perimeter of the bread slice?</p> <p>Example: If the perimeter of a rectangular kitchen sponge is 22 inches and one side is 4 inches, what are the missing side lengths?</p> <p>Example: The area of both our habitat-study plots is equal — 100 cm square — but the perimeter lengths are different. What are two different perimeters that produce this area? (20 cm X 5cm = 10 cm X 10cm)</p> <ul style="list-style-type: none"> • Traveling a Mathematical Loupe Area and Perimeter p. 176 • Thoreau's backpack p.177 • A Yard of Yard, p. 148, and Habitats / Systems



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Geometry	3.G
<p>Reason with shapes and their attributes.</p> <p>3.G.1</p> <p>1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> <p>3.G.2</p> <p>2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i></p>	<p>Reason with shapes and their attributes.</p> <p>2. Students begin with loupe-study objects or locations and partition the shapes into parts with equal areas. They express the area of each part as a unit fraction of the whole. The teacher creates partitioning challenges for students, then students create partitioning challenges for each other to solve. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i></p> <p>Examples:</p> <ul style="list-style-type: none">- If you divide a bread slice into 4 equal parts, what fraction of the area is each part? (1/4th).- Partition the terrarium's floor into 8 equal areas. What fraction is each part? (1/8th) <ul style="list-style-type: none">• Traveling a Mathematical Loupe Area and Perimeter p. 176• Traveling a Mathematical Loupe: Nature's Geometry, p.176• Geometry in Nature: The Hexagon Kit, p. 184• Symmetry Loupe Hunt, Asymmetry, p. 185• Tessellations, p. 190-191